Numerical modeling of self-focusing beams in fiber amplifiers

Roger L. Farrow,[1]* G. Ronald Hadley[2], Arlee V. Smith[2], and Dahv A.V. Kliner[1]
[1]Sandia National Laboratories, P.O. Box 969, MS 9056, Livermore, CA 94551
[2]Sandia National Laboratories, P.O. Box 5800, Albuquerque, NM 87185
*e-mail: farrow@sandia.gov, phone: (925)294-6136, FAX: (925)294-2595

ABSTRACT

We have numerically investigated the behavior of the LP_{01} fundamental mode of a step-index, multimode (MM) fiber as the optical power approaches the self-focusing limit (P_{crit}). The analysis includes the effects of optical gain and fiber bending and are thus applicable to coiled fiber amplifiers. We find that at powers below P_{crit}, there exist stably propagating power-dependent modifications of the LP_{01} mode, in contrast to some previous solutions that exhibited large-amplitude oscillations in beam waist along the fiber. For the first time, to our knowledge, we show that in a MM fiber amplifier seeded with the low-power LP_{01} eigenmode, the transverse spatial profile will adiabatically evolve through power-dependent stationary solutions as the beam is amplified toward P_{crit}. In addition, for a given value of the nonlinear index, P_{crit} is found to be nearly the same in the bulk material and in a step-index fiber. These conclusions hold for both straight and bent fibers, although the quantitative details are somewhat different.

Keywords: self-focusing, fiber amplifiers, high-power fiber amplifier, self-focusing in fibers

1. INTRODUCTION

An upper limit to the attainable beam power in a fiber amplifier is imposed by self focusing (SF), which arises from irradiance-dependent contributions to the refractive index. Although SF in fibers has been studied for some time, significantly different conclusions are found in the literature concerning the ultimate SF limit and the nature of beam propagation as P_{crit} is approached. For example, authors have predicted both periodic variations in beam diameter along the fiber[1,2] and non-oscillatory solutions[3] at optical powers approaching P_{crit}. In addition, there has been disagreement over the value of P_{crit} for a waveguide compared to a bulk dielectric.[4,1] Furthermore, relevant to high-power limitations of fiber amplifiers, no SF studies of coiled fibers or fibers with gain have been reported, to our knowledge.

These issues are critical in determining practical limits to power scaling in fiber lasers. If high-power beams are indeed destined to oscillate strongly in diameter when approaching P_{crit}, then the *practical* SF limit on power will be significantly lower than that expected for a bulk dielectric. Even below breakdown, energy extraction efficiency and nonlinear thresholds would be reduced in the regions of smallest beam waist. Moreover, the beam profile in a fiber that is coiled for mode-filtering[5] or for packaging can be substantially displaced, compressed, and can lose cylindrical symmetry; SF of such beams might be expected to vary significantly from the undistorted mode, but has not been analyzed. Finally, to our knowledge, the behavior of a beam undergoing amplification toward P_{crit} has not been examined.

In this paper we address the following questions:

(1) What are the effects of amplification on the propagation of eigenmodes of a step-index fiber at optical powers approaching P_{crit}? In particular, are longitudinal oscillations in beam diameter necessarily present?

(2) How does coiling of the fiber affect the propagation of amplified and unamplified beams at powers approaching P_{crit}?

(3) How does P_{crit} compare between straight and coiled step-index fibers and a similar bulk material?

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Our calculations are not restricted to the paraxial approximation, and our methods are applicable to fibers with arbitrary refractive-index profiles, with gain and/or loss, and with any bend radius. We make no assumptions concerning the spatial profile of the beam.

2. NUMERICAL SIMULATIONS

We performed numerical studies of high-power beams propagating in both straight and coiled step-index fibers with Kerr-type nonlinearity using two numerical methods. A beam propagation method (BPM) was used to study the propagation of a specified input field distribution through straight or bent fibers with or without power gain. We also employed an eigenmode solver[6] to find stably propagating solutions at a specific beam power in either straight or bent fibers, using an interative technique described below. The solver was also used to obtain normal eigenmodes (which did not include nonlinear index contributions) for use as input fields in some cases.

Both codes were based on a triangular-mesh finite-difference scheme[6] in which triangular boundaries are made to lie along all dielectric interfaces. A time-independent, vectorial Helmholtz equation including Kerr nonlinearity is expressed in cylindrical coordinates describing a toroidal geometry appropriate for bent fibers. In developing finite-difference expressions for computer solution, we retained bending terms to higher order than the commonly used effective refractive-index method.[7] We found that semivectorial results (neglecting the small-field component) were sufficiently accurate for the low numerical apertures (NAs) typical of large-mode-area (LMA) fibers. In the BPM code we implemented a wide-angle beam propagation methodology, obtaining a propagation matrix that is solved using standard linear algebra techniques. For the eigensolver, which used the same triangular mesh and finite difference expressions to approximate the Helmholtz equation, we solved the resulting generalized eigenproblem using a iterative algorithm based on the implicitly restarted Arnoldi method.[8]

For concreteness, the fiber specifications were chosen to match those used in previous fiber amplifier experiments,[5,9] *i.e.*, a core radius (a) of 12.5 μm, core refractive index of 1.453125, core NA of 0.10, and free-space wavelength (λ) of 1064 nm, resulting in a V-number of 7.38 (corresponding to a MM fiber that supports ~26 modes). We assumed the nonlinear index coefficient n_2 to be equal in the core and cladding, and we used the literature value for fused silica (2.7×10^{-16} cm^2 W^{-1}).[10] This value yields P_{crit} = 4.36 MW for a Gaussian beam in bulk material, using an expression derived by Fibich and Gaeta.[4] For the high-power beam in our calculations, we chose a representative power of 3 MW. For the bent fiber we assumed a 0.84-cm radius of curvature[5] corresponding to a power loss coefficient of 6.7 dB m^{-1} for LP_{01}.

3. RESULTS

3.1. Stable, high-power propagation in straight fibers

In addressing question (1), we examined whether a beam will necessarily undergo spatial oscillations as it is amplified to powers approaching P_{crit} in the nonlinear fiber. Using the BPM code, the LP_{01} (normal) eigenmode of the straight fiber was launched at a power P = 1 kW << P_{crit}. To reduce run times, we amplified in discrete stages, with gain coefficients up to 2700 dB/m used for initial amplification where SF effects were negligible. The gain was longitudinally uniform but reduced in two or three stages to ≈700 dB/m to ensure adiabatic propagation near P_{crit}. Plotted in Fig. 1 is the $1/e^2$ beam radius *vs.* propagation distance, showing a relatively smooth decrease due to SF and a lack of oscillations. Only the final cm before catastrophic SF is shown in the plot, but no significant oscillations were observed in earlier propagation. Fig. 1 also shows the beam power determined from transverse integration of the solution irradiance. The latter has reduced accuracy below 2 μm in radius, explaining the deviation from exponential gain at the end of propagation. The smoothly varying beam waist demonstrates that amplification of a low-power eigenmode does not necessarily result in oscillations, even at gains much higher than possible in actual fiber amplifiers (typically <60 dB/m).

Transverse profiles through the center of the fiber, taken from the above results, are plotted in Fig. 2 for various beam powers. The profiles narrow with increasing power as a result of SF, and they develop extended wings qualitatively similar to Lorentzian profiles. We investigated whether the transverse field

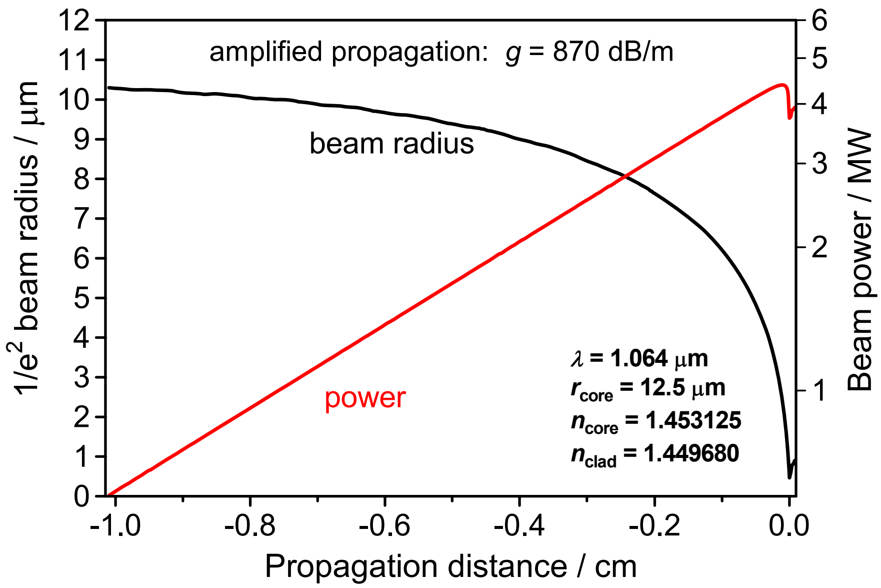

Figure 1. The fundamental mode of an LMA fiber was amplified from an initially low power (not shown) using a beam-propagation method (BPM) code that treats Kerr nonlinearity in the refractive index. Beam radii and total power are shown as a function of distance from catastrophic SF.

distribution at a particular beam power could propagate stably at that power by launching the field with the same fiber parameters but without gain. The solid blue curve in Fig. 3 shows the resulting beam-waist dependence on propagation for the 3-MW field distribution. Only a small periodic modulation, attributed to numerical approximations of the BPM code, is observed, demonstrating that the adiabatically amplified field corresponds to a longitudinally invariant solution of the nonlinear fiber for P = 3 MW. (The fields amplified to other powers ≤ 3.6 MW behaved similarly. We also generated video clips of the field irradiance distributions versus distance and confirmed visually that the entire 2-dimensional profiles were stable.) For comparison, the beam-waist dependence of the low-power LP_{01} eigenmode launched with 3 MW is shown by the red curve. This large modulation is comparable to that of the calculations of Ref. 2 for an eigenmode launched at the same power. It is important to note that the oscillations would not occur in an actual fiber amplifier in which the low-power LP_{01} mode was amplified toward P_{crit}. From comparing the grey curve labeled as 0 MW in Fig. 2 (essentially the LP_{01} eigenmode profile) with the profile of the 3-MW beam, it is clear that significant differences exist in the transverse field distribution of the eigenmode and that of the stably propagating solution for P near P_{crit}.

3.2. Uniqueness of solutions

We investigated the uniqueness of the stable solutions by seeking additional solutions using the eigensolver with an iterative method. Beginning with an eigenmode of the unperturbed fiber, we added n_2 contributions to the fiber index profile according to the modal irradiance, and then re-solved for the eigenmode until successive solutions converged. These solutions are not true eigenmodes of the nonlinear waveguide because they depend on the irradiance, but they individually exhibit mode-like behavior, *i.e.*, a transverse profile that propagates without change. The longitudinal invariance of the solution at 3 MW found by this method is indicated by the beam-waist dependence plotted with circles in Fig. 3, the stability of which is seen to be similar to that of the adiabatically amplified solution. Moreover, the transverse profiles obtained by the two methods agree closely, as shown by the blue curve and circles in Fig. 4. This result strongly suggests that the longitudinally invariant solution evolved from a given eigenmode is unique for a given power. In investigating other eigenmodes, *e.g.*, LP_{11} and LP_{21}, we found that stable solutions also exist and are in close agreement when found using both methods.

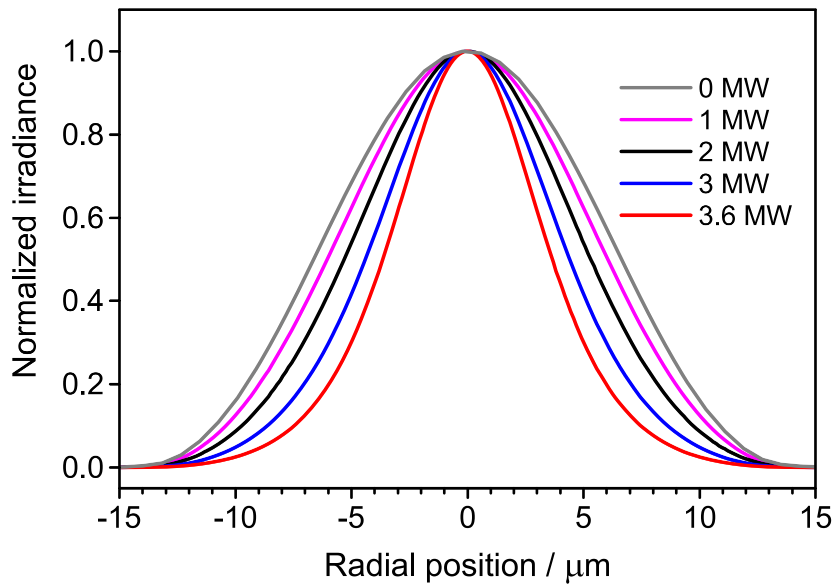

Figure 2. Comparison of transverse profiles of the amplified beam from the calculations of Fig. 2 at the indicated beam powers. The narrowing of the profile results from SF.

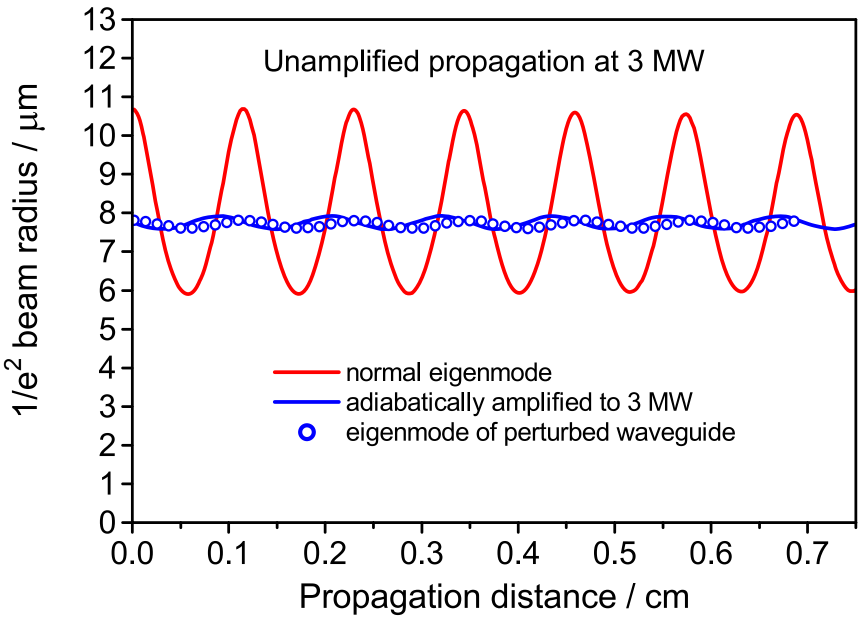

Figure 3. Variation of beam waist with unamplified propagation at 3 MW power for the following input beams: the normal LP_{01} eigenmode of the waveguide, the LP_{01} eigenmode previously amplified to 3 MW (profile plotted in Fig. 2), the LP_{01} eigenmode of the waveguide with nonlinear index contributions included (determined in an iterative calculation).

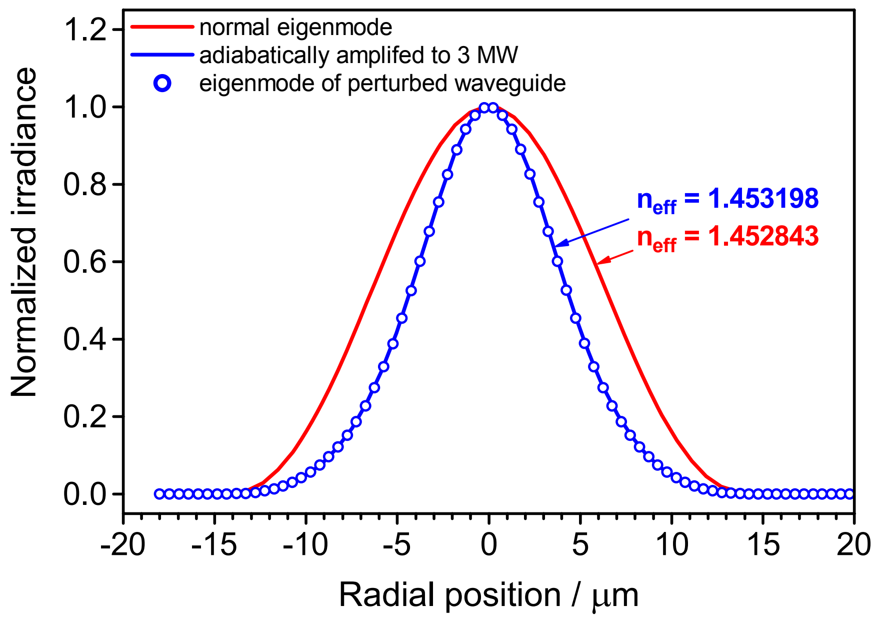

Figure 4. Comparison of transverse profiles of the input beams used in the calculations of Fig. 3: normal LP_{01} eigenmode, the LP_{01} eigenmode previously amplified to 3 MW, and the eigenmode of the waveguide with nonlinear index contributions included (determined in an iterative calculation).

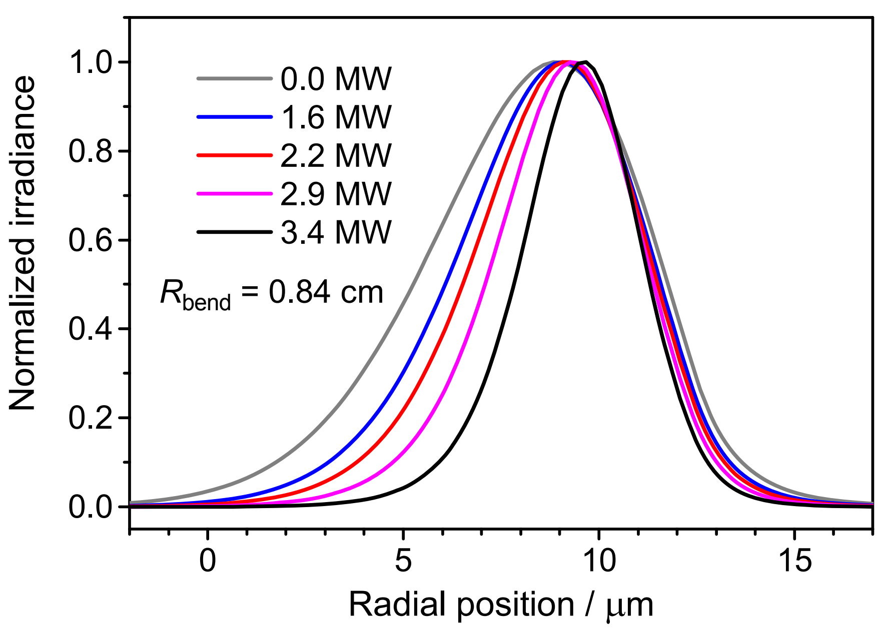

Figure 5. Comparison of transverse profiles of an amplified beam propagating in a bent fiber, at the indicated beam powers. A power gain coefficient of 6.7 dB/m was used. The narrowing of the profile results from self-focusing.

3.3. Coiled fibers

To address question (2) we performed calculations similar to those of Figs. 1-4 but using the coiled-fiber parameters. Again, a smoothly decreasing beam waist was observed during the final stage of amplification of the LP_{01} eigenmode from 100 W to ≈ 4 MW. Plots of the transverse beam profiles through the center of the fiber in the bend plane are shown in Fig. 5. The profiles are asymmetric and shifted significantly from the core center as a result of fiber bending.[6,11] It is interesting to note that the asymmetry has nearly vanished in the profile at the highest power. Again, we verified that that the amplified fields were longitudinally invariant solutions by propagating a selected field (3 MW; not plotted) using the same fiber parameters without gain. The resulting beam-waist stability is illustrated by the blue curve in Fig. 6. The corresponding solution obtained using the iterative method resulted in similar stability (blue circles). Propagation of the eigenmode at 3 MW yielded oscillations (red curve) that are reduced in amplitude compared to the straight fiber but exhibit more than one frequency.

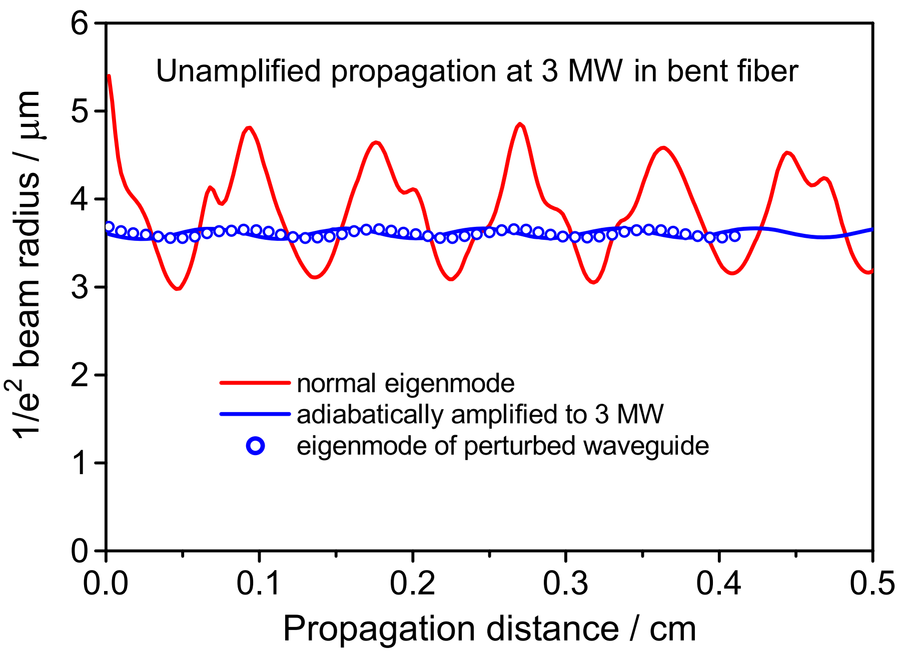

Figure 6. In the bent fiber, variation of beam waist with unamplified propagation at 3 MW for the following input beams: the normal LP_{01} eigenmode of the waveguide, the LP_{01} eigenmode previously amplified to 3 MW, the LP_{01} eigenmode of the waveguide with nonlinear index contributions included (determined in an iterative calculation).

3.4. Higher-order modes

We similarly investigated higher-order LP_{11} and LP_{21} modes in straight and bent fibers and found high-power solutions analogous to those for LP_{01}. Fig. 7a shows comparisons of irradiance profiles of the normal eigenmode (red curve) and high-power solution for LP_{11} at $P = 3$ MW (blue curve). Shown in Fig. 7b are profiles of the LP_{21} eigenmode (red curve) and high-power solution (blue curve) at the same power. Compared to the fundamental mode, profiles of these higher-order modes were less affected by SF; *i.e.*, the stably propagating profiles more closely resembled the eigenmode profiles. In addition, we found that propagation of the higher-order eigenmodes at high power exhibited significantly less modulation of the effective beam radius, R_{eff} (defined as $R_{\mathrm{eff}} = (A_{\mathrm{eff}}/\pi)^{1/2}$ in relation to the effective area) in comparison to the fundamental mode. It seems reasonable to attribute the reduced influence of SF in these modes to the multilobed nature of the LP_{x1} modes, which consist of $x + 1$ separate lobes (see contour plots in Fig. 7). Assuming that the lobes are non-interacting with respect to nonlinear index contributions, each could be viewed as a separate beam, and to the extent that they behaved similarly to an LP_{01} profile, would require $x + 1$ times as much total power to exhibit the same degree of SF. For example, one might expect that P_{crit} would be increased by this factor. However, this proved not be the case as discussed below.

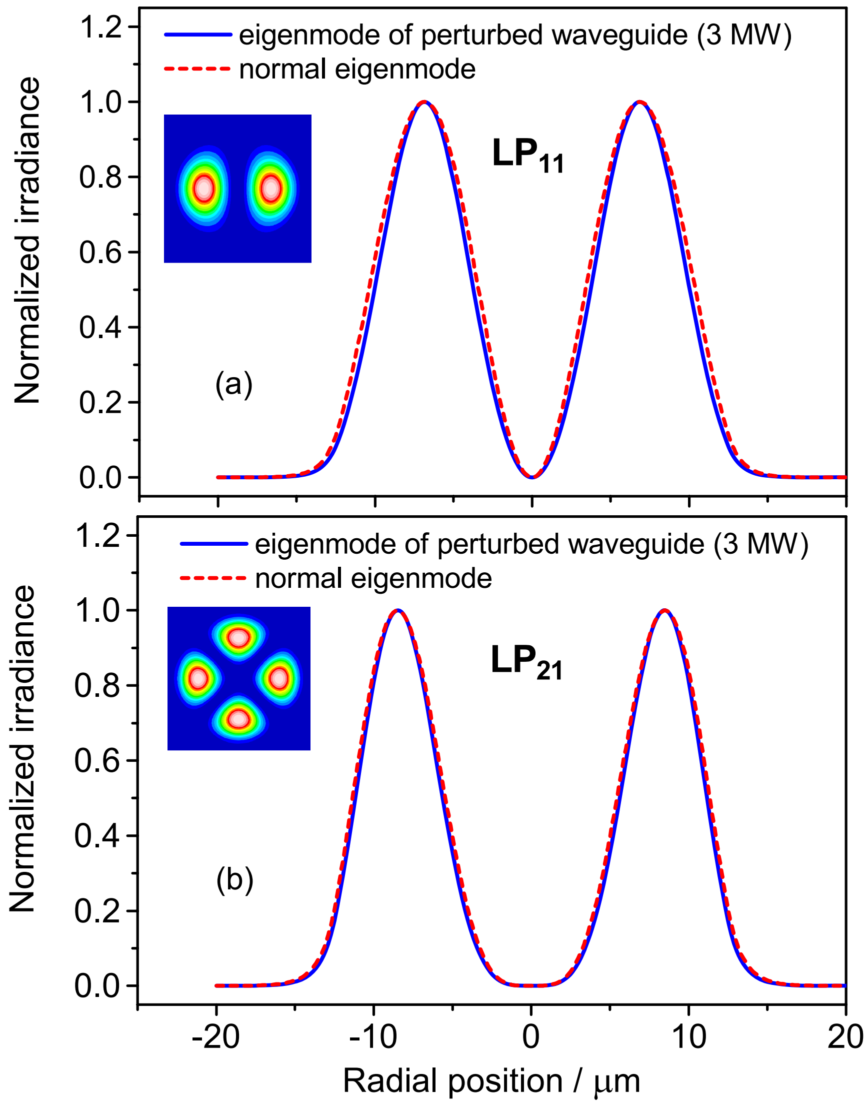

Figure 7. Comparison of horizontal transverse profiles of the normal LP_{11} and LP_{21} eigenmodes and longitudinally invariant solutions at 3 MW. Here the solutions corresponded to eigenmodes of the waveguide with nonlinear index contributions included, determined iteratively. Contour plots of the eigenmode irradiances are shown for reference; the horizontal and vertical axes are 30 μm wide.

3.5. Critical power

We investigated P_{crit} values in the straight and coiled fiber and in a bulk medium with the same value of n_2. P_{crit} is defined as the lowest power at which a beam undergoes catastrophic self-focusing, *i.e.,* focuses to an infinitesimal waist. For calculations involving a finite spatial resolution, Fibich and Gaeta[4] adopted a phenomenological definition for P_{crit} as the minimum power at which the focused peak irradiance reached 10^4 times the initial peak irradiance. This criterion was supported by showing that the numerically calculated value in a gas-filled waveguide was equal to the analytical value for a Townes soliton (4.29 MW for our material parameters).[4] We defined P_{crit} as the lowest power at which the beam radius (e^{-2}) decreased to less than w_{SF} = 1 μm, corresponding to loss of resolution by the calculation mesh. As with the above phenomenological definition, this approach provides a P_{crit} value that represents the SF limit in any practical device. To determine P_{crit} we launched a high-power solution (representative power of 3 MW) into the fiber at a series of beam powers *greater than* 3 MW, typically in steps of 0.125 MW. Because the input field distribution was not a stationary mode at the input power, the beam profile underwent periodic oscillations as it propagated without amplification in the fiber. P_{crit} was taken as the lowest power in which the initial beam waist was < w_{SF}. For the bulk medium we launched a Gaussian profile beam from an initial waist of 8 μm; the beam diverged for all powers below a threshold power ≡ P_{crit}, above which it focused to a waist < w_{SF}.

In the straight fiber, we obtained a value of 4.3 MW for P_{crit} of LP_{01}, in agreement with the waveguide results of Ref. 4. For the bulk dielectric, P_{crit} for an initial Gaussian profile was also found to be 4.3 MW, in close accord with the 4.36 MW value from Ref. 4. We conclude that the SF limit is not significantly changed by the fact that the beam is propagating in a nonlinear waveguide instead of a bulk material, in contrast to Ref. 1 but in agreement with Ref. 4. For a coiled fiber, P_{crit} was found to be ≈ 4.2 MW, essentially the same as in the straight fiber.

The self-focusing behavior of the higher-order modes differed significantly from that of the fundamental mode. When the latter was adiabatically amplified from low power up to P_{crit} ≈ 4.3 MW, the beam waist decreased to w_{SF} (see Fig. 1). However, when similarly amplified, the LP_{11} and LP_{21} modes were able to sustain higher beam powers before SF occurred, 8.5 MW and 17 MW respectively. Thus, it appeared that these modes had correspondingly higher values of P_{crit}. In addition, the field distributions at powers below the onset of SF (but well above P_{crit} of the fundamental mode) appeared to be capable of stable propagation at these powers. However, we found that extended propagation distances, typically greater than 0.8 cm, revealed the onset of an instability in the form of erratic, oscillatory power transfer between the lobes. When the oscillations increased to the point that most of the power transferred into one of the lobes, SF would occur shortly afterward. The instability occurred at beam powers as low as 5 MW and 6 MW for the LP_{11} and LP_{21} modes, respectively. Thus, it appears the practical P_{crit} values for these modes are not significantly greater than for LP_{01}.

SUMMARY

In summary, we have found that:

(1) At optical powers approaching P_{crit} in step-index fibers, power-dependent solutions exist that propagate without change. Light injected with other spatial profiles or powers will exhibit oscillatory behavior. In a MM fiber amplifier seeded with the low-power LP_{01} eigenmode, the field distribution adiabatically evolves into that of the fundamental stationary solution as the beam is amplified toward P_{crit}. We observed similar adiabatic evolution into stationary solutions when amplifying the higher modes LP_{11} and LP_{21}. However, for a given power, differences between the solutions and the initial mode profiles were less pronounced than for the fundamental mode.

(2) These conclusions hold for fibers that are either straight or coiled, although the quantitative details are somewhat different.

(3) For a given value of n_2, P_{crit} is nearly the same in the bulk material and in a step-index fiber. For bulk fused silica and for a straight fiber, we obtained the same value, 4.3 MW, and for a bent fiber we obtained 4.2 MW (the same within our estimated relative uncertainty of 0.2 MW). It should be noted that these values are expected to depend approximately inversely[4] on the assumed value of n_2. The higher-order, multi-lobed modes LP_{11} and LP_{21} behaved substantially differently above P_{crit} compared to LP_{01}. The former underwent catastrophic SF only after an onset of instabilities involving the distribution of power among the lobes; the latter did not exhibit instabilities leading up to SF.

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