

Fig. 7. Small signal gain of mode two versus frequency offset ( $\nu_2 - \nu_1$ ) for different  $z$  positions along the fiber. The laser gains are included and can be found from the left axis intercepts.

location are taken from the curves of Fig. 5. The shapes of the curves of gain versus frequency at  $z=0$  and  $z=1$  m are well fit by functions with the form of Eq. (6), and with the peak at approximately the predicted frequencies based on the effective lifetimes.

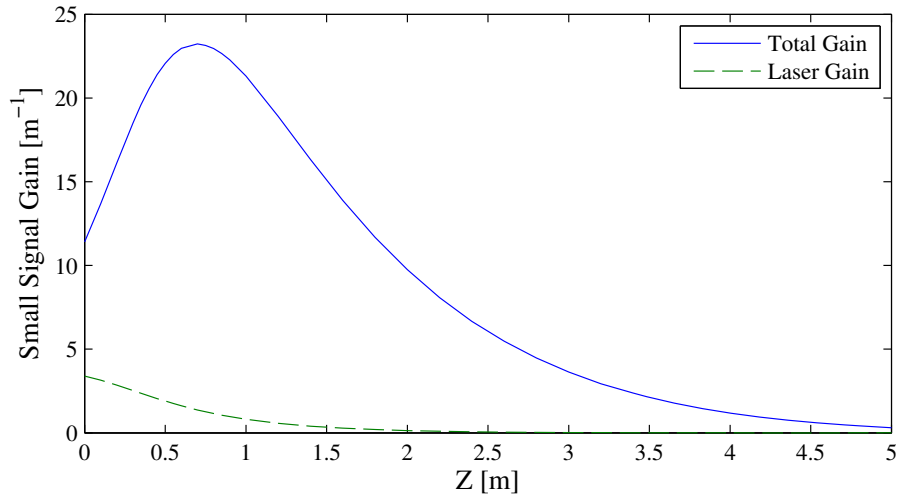


Fig. 8. Small signal gain of mode two at 200 kHz for the test amplifier. The solid curve is total gain of mode two; the dashed curve shows the contribution of laser gain.

Assuming the power in mode 2 remains small, we can express it as

$$P_{s2}(z) = P_{s2}(0)e^{G(z)} \quad (16)$$

where  $G(z)$  is the integral of the small signal gain,

$$G(z) = \int_0^z g(z')dz'. \quad (17)$$

Summing the gain curves of Fig. 7 gives an approximation of  $G(L)$  that indicates the frequency offset that maximizes  $G(L)$  lies near 200 kHz. We computed the small signal gain  $g(z)$  at 200 kHz for a fine set of  $z$  intervals and display the result in Fig. 8. Finally, we ran the KK model over an extended length of fiber with the usual launched powers of 1200 W in the pump and 30 W in mode one, plus 1  $\mu$ W in mode two. The model results are shown in Fig. 9.

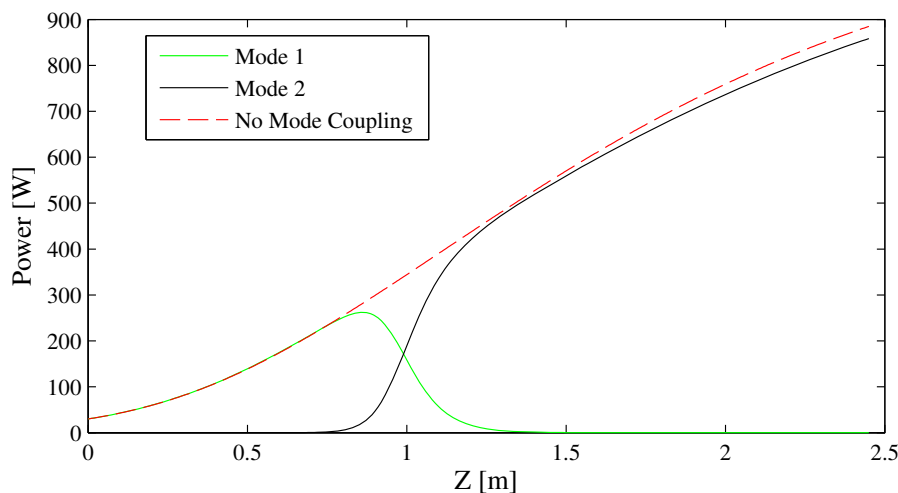


Fig. 9. KK coupling at 200 kHz with inputs of [1200, 30, 1E-6] W for [pump, mode one, mode two]. The dashed curve shows mode one power without mode coupling.

Initially mode two is weak, and mode one is amplified just as though there were no mode coupling, but after approximately one meter the power is abruptly transferred from mode one to mode two where it remains thereafter. The amplification of mode two is slightly less than that of mode one beyond one meter because of poorer overlap of mode two with the doped core, so the power in mode two falls slightly below the reference curve for mode one. The light that is transferred from mode one to mode two is also frequency shifted to match the frequency of mode two. As a caution we note that the value of the KK coupling coefficient is quite uncertain, and the value we have used may be too large by an order of magnitude or more. However, this model serves to illustrate mode coupling, and the chosen coupling coefficient leads to a mode coupling strength similar to the thermal coupling described next.

## 7. Thermal mode coupling model results

The thermal coupling coefficient is much better known than the KK coefficient so we are confident that the thermally induced gains presented here are quite accurate. Figure 10 shows the small signal gain of mode two versus the frequency difference ( $\nu_1 - \nu_2$ ) at several positions along the test amplifier. Near the input the gain peaks near 2.5 kHz, and the peak shifts to approximately 2 kHz at one meter and later. Unlike the corresponding curves for KK coupling (Fig. 7), the curves near the input end are not well fit by the form in Eq. (6). This is not surprising because the phase delay in this case is not local as in KK, but is due to thermal diffusion. The thermal model runs much slower than the KK model because integrating the thermal diffusion equation requires fine time steps for convergence. The maximum time step is fixed by the x,y dimensions and the diffusion constant, so the computation time and the required memory increase with a decreasing frequency offset, making the computation especially slow for frequency offsets less than a few kHz. For this reason we have not attempted to propagate over

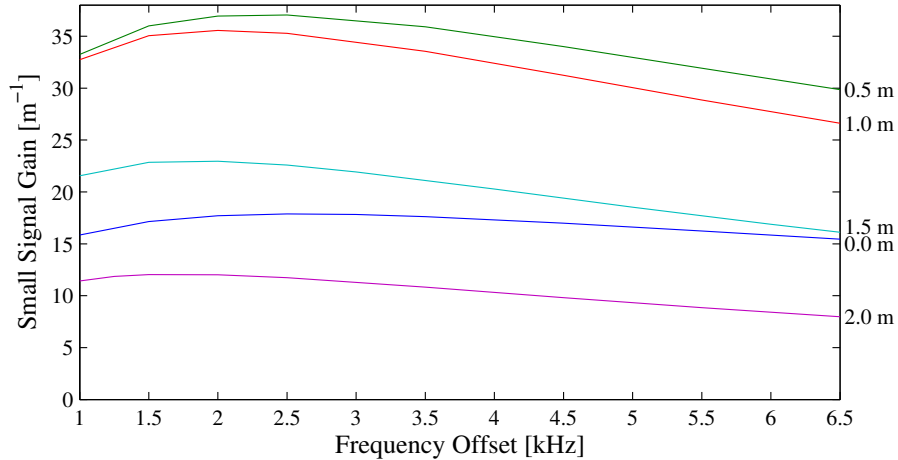


Fig. 10. Small signal gain of mode two versus frequency offset ( $\nu_1 - \nu_2$ ) for different  $z$  positions along the fiber. The laser gain is included but it is a small fraction of the small signal gain.

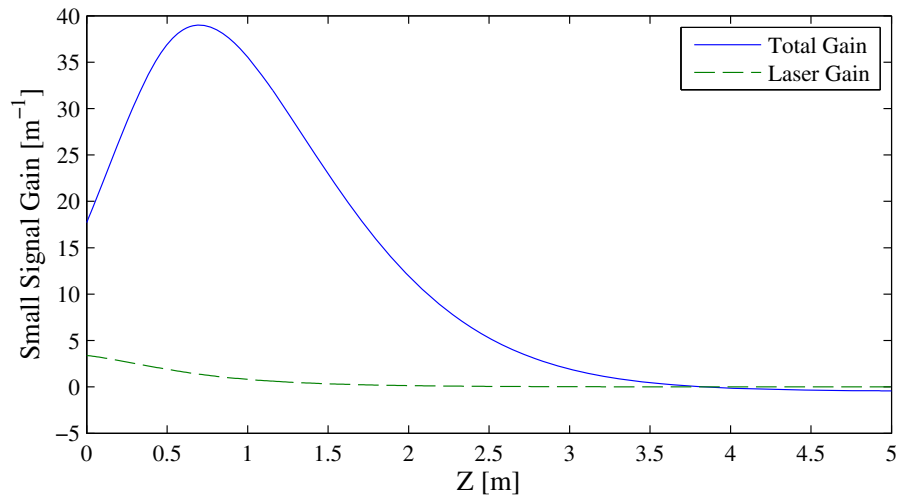


Fig. 11. Small signal gain of mode two at 2 kHz versus  $z$  for the test amplifier. The solid curve is total gain of mode two and the dashed line is the contribution of laser gain. The mode coupling gain is the difference between the two curves.

a long fiber section as we demonstrated using the KK model, but we can compute the small signal gains by propagating over short distances. We propagate a distance of one beat length  $L_{12}$  or half that, because at these distances all times in the full time cycle experience identical gains. A similar procedure with the KK model gave good agreement between the net gain calculated by integrating the small signal gains computed from short propagation lengths, and the net gain computed directly by propagating over the full distance. Figure 11 shows the small signal gain of mode two due to thermal coupling at a frequency offset of 2 kHz. This frequency offset was chosen to maximize the net gain along the fiber, based on the curves in Fig. 10. The slightly negative gain near the output end an artifact of the short propagation distances used in

computing the gain.

We modeled the test amplifier with pump powers of 600 W and 900 W in addition to 1200 W, and plot the integrated gain coefficients  $G(L)$  as symbols in Fig. 12. The value of  $G(L)$  is nearly linear in the pump power, so the total gain of mode two grows exponentially with the pump power. This will result in a sharp threshold in pump power for mode degradation via thermally induced transfer of light from mode one to mode two.

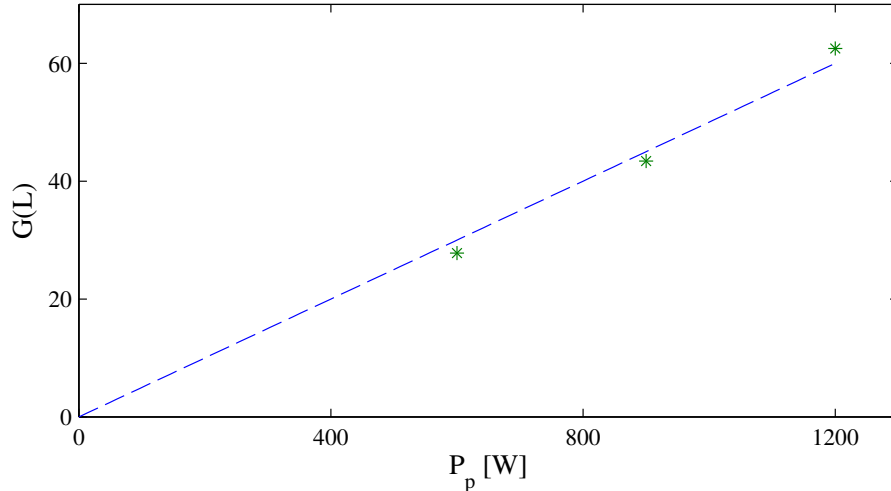


Fig. 12. Net gain of thermal mode coupling versus pump power. The symbols are the integrated small signal gains of mode two for three pump powers. The dashed line is for comparison with a linear dependence on pump power.

Comparison of the KK and thermal gain curves in Figs. 9 and 11 reveals that thermal mode coupling is much stronger than KK mode coupling near the amplifier input end, but falls more rapidly near the output end. The integrated small signal gain for KK coupling at 200 kHz is  $G(L) = 44.3$ ; that for thermal coupling at 2 kHz is  $G(L) = 62.5$ , so thermal mode coupling is stronger than KK coupling even for a KK coefficient that is perhaps too large.

## 8. Conclusions

Our model makes several simplifying assumptions. For example, we assumed only modes one and two are populated, and the light in each is monochromatic with a fixed frequency separation. Nevertheless, we think our results strongly indicate that thermal, and perhaps KK, mode coupling can cause mode degradation in laboratory amplifiers. In reality other modes will also be coupled to modes one and two. This can introduce mode coupling losses to mode two that will partially counteract the gain we have computed. This power flow to additional modes may add to the mode degradation, and given the extremely high gains we have computed, the power flow to a variety of modes could be substantial.

Our assumption of monochromatic light is not limiting either. Our results should apply to broadband light in modes one and two if they have frequency components with the right frequency offsets for high gain. The gain bandwidth for mode coupling is quite broad relative to the frequency at maximum gain, so components within a broad range can be amplified. The expected result is fluctuating mode coupling and mode degradation. A Fourier transform of the fluctuations would provide the spectrum of amplification.

Our assumed initial condition of almost all of the light is in mode one with a small amount

seeding mode two raises the question of the source of the mode two seed light. Its origin is not entirely clear. The weak seed could be created by mechanical vibration of the fiber, particularly at the fiber input where coupling into the fiber is highly position sensitive, or it could be created by oscillations in the pump power or spectra, or by fluorescence from the upper laser level, or it might be present in the injected signal light.

Detailed experimental studies will be necessary to verify our model and to identify sources of the mode two seed light. However, our model for thermal mode coupling appears to agree at least qualitatively with certain reported behaviors of laboratory amplifiers: a sharp pump threshold for mode degradation is predicted and reported; the pump power threshold is in the observed range; the frequency offset of 2-3 kHz for maximum gain agrees with reported fluctuation times; the mode coupling gain occurs primarily in the first half of the amplifier which is consistent with observations that coiling the first half of the amplifier often suppresses mode degradation.

If our model can be more fully validated, it should prove useful in improving amplifier designs in order to raise the mode degradation threshold. For example, we can apply it to bent fiber and to fiber with non-step index profiles or non-step Yb doping profiles. It can be used to compare co-pumped and counter-pumped amplifiers and different pump spectra. We can systematically study trends with core to pump cladding size ratios, and the length variations this implies.

### **Acknowledgments**

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