# Peak-power limits on fiber amplifiers imposed by self-focusing

**Roger L. Farrow and Dahv A. V. Kliner**

*Sandia National Laboratories, Livermore, California 94551*

**G. Ronald Hadley and Arlee V. Smith**

*Sandia National Laboratories, Albuquerque, New Mexico 87185*

Received June 12, 2006; revised August 10, 2006; accepted August 22, 2006;
posted September 13, 2006 (Doc. ID 71859); published November 9, 2006

We have numerically investigated the behavior of the fundamental mode of a step-index, multimode (MM) fiber as the optical power approaches the self-focusing limit ($P_{crit}$). The analysis includes the effects of gain and bending (applicable to coiled fiber amplifiers). We find power-dependent, stationary solutions that propagate essentially without change at beam powers approaching $P_{crit}$ in straight and bent fibers. We show that in a MM fiber amplifier seeded with its fundamental eigenmode at powers $\ll P_{crit}$, the transverse spatial profile adiabatically evolves through a continuum of stationary solutions as the beam is amplified toward $P_{crit}$. © 2006 Optical Society of America

*OCIS codes:* 190.4370, 060.4370, 260.5950, 140.3510.

Dramatic increases in the output power of rare-earth-doped fiber lasers and amplifiers are allowing these sources to displace conventional lasers in a variety of applications.[1] A widely used method for power scaling of fiber sources is bend-loss-induced mode filtering, in which a highly multimode (MM), large-mode-area (LMA) fiber is coiled with a radius of curvature chosen to introduce substantial loss for high-order modes and relatively little loss for the fundamental mode ($LP_{01}$).[2,3] For standard step-index fibers, this coiling results in substantial distortion of the fiber modes, including the loss of azimuthal symmetry.[4,5]

An upper limit to the attainable peak power from pulsed fiber sources is imposed by self-focusing (SF) in the fiber. Theoretical studies have investigated beam propagation in fibers at powers approaching catastrophic SF ($P_{crit}$). Authors have reported both oscillatory solutions, characterized by periodic variations in beam diameter along the fiber, and nonoscillatory, stable solutions for step-index,[6] parabolic-index,[7,8] and gas-filled, hollow-core fibers.[9] Two publications also reported only oscillatory solutions for step-index[10] and gas-filled, hollow-core[11] fibers. Such oscillations would effectively lower the threshold power for the onset of the nonlinear process and optical damage and would thus be of great practical significance. Also of practical interest, Fibich and Gaeta[12] calculated that $P_{crit}$ is similar in a waveguide and a bulk material, whereas Ref. 11 reported a substantially higher value (by a factor of 5) in the waveguide. All of these analyses were performed for fibers without gain (i.e., not for fiber amplifiers) and for straight fibers rather than the coiled fibers employed in practical systems.

In the present paper, we address the following questions:

(1) What are the effects of amplification on the propagation of the fundamental mode of a step-index fiber at optical powers approaching $P_{crit}$? In particular, are longitudinal oscillations in beam diameter necessarily present?

(2) How does coiling of the fiber affect the propagation of amplified and unamplified beams at powers approaching $P_{crit}$?

(3) How does $P_{crit}$ compare between straight and coiled step-index fibers and a similar bulk material? Our calculations are not restricted to the paraxial approximation, make no assumption with respect to spatial beam profile, and are applicable to fibers with gain or loss, arbitrary refractive index profiles, and any bend radius.

We simulated the propagation of a high-power beam in step-index fibers (straight and bent) using a beam propagation method (BPM) based on a triangular-mesh finite-difference scheme.[4] A time-independent Helmholtz equation including Kerr nonlinearity is expressed in cylindrical coordinates describing a toroidal geometry appropriate for bent fibers. The semivectorial finite-difference expressions (appropriate for LMA fibers with low numerical apertures) included bending terms to higher order than the commonly used effective refractive index method,[13] and employed a wide-angle BPM. An eigenmode solver[4] based on essentially the same field equations and finite-difference scheme was also used as described below.

For concreteness, the fiber specifications were chosen to match those used in previous MM amplifier experiments,[2,3] i.e., a core radius ($a$) of 12.5 $\mu$m, a core refractive index of 1.453125, a core NA of 0.10, and the free-space wavelength ($\lambda$) of 1064 nm, resulting in a V number of 7.38 ($\sim$26 guided modes). We assumed $n_2$ to be equal in the core and cladding, and we used the literature value for fused silica ($2.7 \times 10^{-16}$ cm$^2$ W$^{-1}$).[14] This value yields $P_{crit}=4.36$ MW for a Gaussian beam in bulk material, using an expression derived by Fibich and Gaeta.[12] For the high-power beam in our calculations, we chose a representative power of 3 MW. For the bent fiber, we assumed a 0.84 cm radius of curvature[3] corresponding to a calculated power loss coefficient of 6.7 dB $m^{-1}$ for $LP_{01}$.

We examined whether a beam will undergo spatial oscillations as it is amplified to powers approaching $P_{crit}$ in the nonlinear fiber. For the case of the bent fiber, the $LP_{01}$ (linear) eigenmode was launched at a power of 80 kW $\ll P_{crit}$. To reduce run times, we amplified in discrete stages, with gain coefficients up to 2700 dB/m used for the initial amplification where SF effects were negligible. The gain was longitudinally uniform but reduced in two or three stages to a minimum of 720 dB/m to ensure adiabatic propagation when approaching $P_{crit}$. Plotted in Fig. 1(a) is the $1/e^2$ beam radius versus the propagation distance, showing a relatively smooth decrease due to SF. Only the final 0.56 cm before catastrophic SF is shown, but no significant oscillations were observed in prior propagation. Figure 1(a) also shows the beam power determined from the transverse integration of the solution irradiance. The latter has reduced accuracy below 2 $\mu$m in radius, explaining the deviation from exponential gain at the end of propagation. Transverse profiles through the center of the fiber in the bend plane are plotted in Fig. 1(b) for various beam powers. The profiles are asymmetric and shifted significantly from the core center as a result of fiber bending.[4,5]

We found that each transverse field distribution is a stationary solution; i.e., if launched into the nonlin-

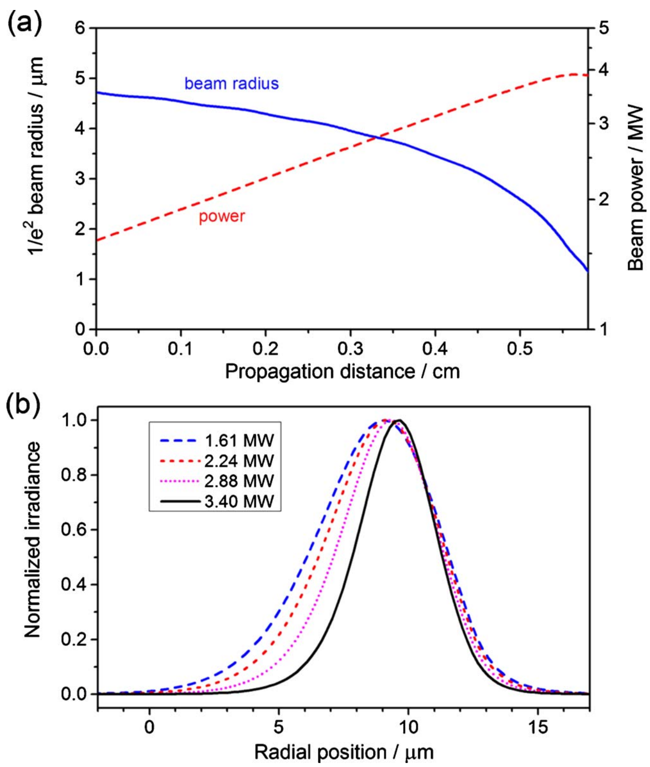


Fig. 1. (Color online) Propagation of an initial $LP_{01}$ eigenmode in a coiled fiber amplifier with a 0.84 cm bend radius for the final 0.58 cm before SF. The gain coefficient was 720 dB/m. (a) $1/e^2$ beam radius (solid curve, left axis) of the transverse profile (through the center of the fiber in the bend plane) and beam power (dashed curve, right logarithmic axis). (b) Transverse profiles along the bend plane for various beam powers.

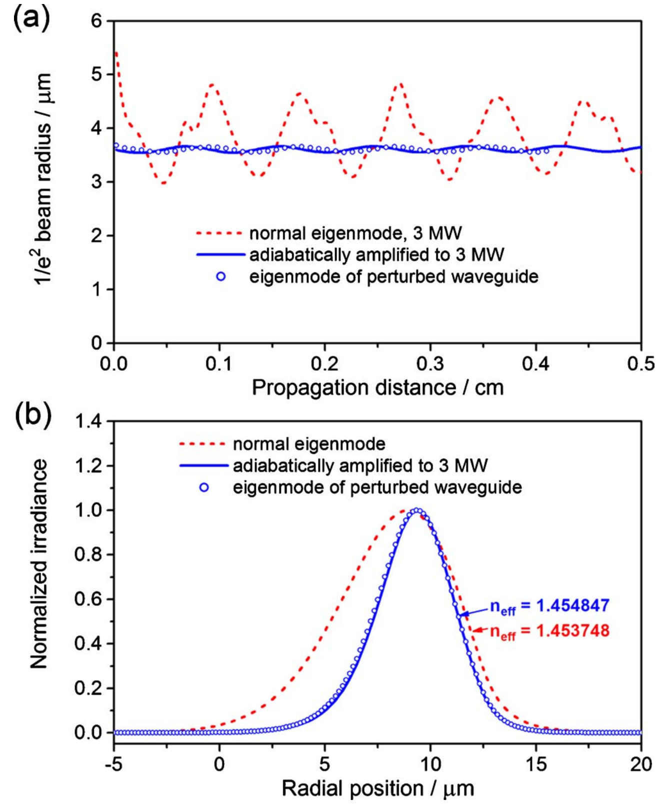


Fig. 2. (Color online) (a) BPM calculations showing the longitudinal dependence of the $1/e^2$ beam radius (along the bend plane) of 3 MW beams propagating unamplified in the coiled fiber. Dashed curve, propagation of the normal eigenmode ($LP_{01}$ of the coiled waveguide). Solid curve and circles, propagation of high-power stationary solutions, whose irradiance profiles are shown in (b); small residual oscillations are a numerical artifact. (b) Spatial profiles of the fundamental $LP_{01}$ mode of a coiled fiber (dashed curve) and of high-power stationary solutions obtained from an iterative technique (circles) and by adiabatic amplification (solid curve intersecting the circles). The effective indices of the fundamental modes shown are denoted $n_{eff}$.

ear fiber at that power, it propagates without change in the absence of gain. For example, Fig. 2(b) shows unamplified propagation of the 3 MW stationary mode with negligible beam waist variation. For comparison, propagation of the linear $LP_{01}$ eigenmode at 3 MW exhibits pronounced oscillations [Fig. 2(b), dashed curve], which are qualitatively similar to those obtained by Igarashi and Galvanauskas[10] in a noncoiled fiber. It is important to note that such oscillations would not occur in an actual fiber amplifier in which the low-power $LP_{01}$ mode was amplified toward $P_{crit}$. We conclude that during amplification, the transverse beam profile adiabatically evolves through a continuum of stationary solutions.

To investigate the uniqueness of the power-dependent stationary solutions, we calculated solutions employing a different approach. The eigenmode code was initially used to find the fundamental mode of the unperturbed waveguide, mode-power-induced refractive index contributions were then added to the latter, and the process was repeated until the solution converged. Figure 2(a) shows the close agreement between transverse profiles for a converged iterated solution and an adiabatically amplified solution, both at a power of 3 MW. The stability of the iterated solution is indicated by its minimal beam-radius variation with propagation [circles in Fig. 2(b)], which are essentially identical to that of the adiabatic solution.

In the beam-propagation calculations, gain coefficients were small enough (720–2700 dB/m) to ensure adiabatic evolution of the mode profile. Gain coefficients in actual fiber amplifiers are typically $\leq$50 dB/m, indicating that the mode profile in a fiber amplifier will propagate adiabatically. For a high-power optical pulse, the transverse spatial profile of the stationary beam is time dependent with respect to power; it would thus be difficult experimentally to launch a stable, high-power pulse. However, such a pulse should grow adiabatically in a short ($\ll$1 km) fiber amplifier for pulses with roughly nanosecond or longer coherence times. These results are of great practical significance for power scaling of fiber sources.

We next compared $P_{\mathrm{crit}}$ values in the nonlinear fiber and a bulk medium. Fibich and Gaeta adopted a phenomenological definition for $P_{\mathrm{crit}}$ (beam intensity $\geq 10^4$ times the input peak intensity) and showed that the numerically calculated value in a gas-filled waveguide is equal to the analytical value for a Townes soliton (4.29 MW for our parameters).[12] For our calculations, we launched the stationary solution for 3 MW into the fiber at various beam powers >3 MW, without amplification. Because the input field distribution was not a stationary mode at the given input power, the beam profile underwent periodic oscillations as it propagated in the fiber, as in Fig. 2(b). We defined $P_{\mathrm{crit}}$ as the power at which the first beam-radius minimum was <1 $\mu$m, corresponding to loss of resolution by the calculation mesh. As with the phenomenological definition in Ref. 12, this approach provides a $P_{\mathrm{crit}}$ value that represents the SF limit in any practical device.

In the straight fiber, we obtained a value of 4.3 MW for $P_{\mathrm{crit}}$, in agreement with the waveguide results of Ref. 12. We also performed SF calculations for a bulk dielectric with the same $n_2$ value as the fiber using a Gaussian input profile with an initial beam radius of 8 $\mu$m. For the uniform dielectric, $P_{\mathrm{crit}}$ was also found to be 4.3 MW, also in close accord with the 4.36 MW value from Ref. 12. We conclude that the SF limit is not significantly changed by the fact that the beam is propagating in a nonlinear waveguide instead of a bulk material, in contrast with Ref. 11 but in agreement with Ref. 12.

SF behavior was qualitatively similar whether the fiber was straight or coiled to a 0.84 cm radius. Both the high-power stationary modes and the linear eigenmodes are distorted by coiling, and the bent-fiber stationary solutions have a somewhat smaller mode-field diameter than those for the straight fiber. $P_{\mathrm{crit}}$ was found to be $\approx$4.2 MW, essentially the same as in the straight fiber.

In conclusion, we have addressed the three questions outlined above. We found that:

(1) At optical powers below $P_{\mathrm{crit}}$, power-dependent stationary solutions exist that propagate unchanged in step-index fibers, in agreement with Refs. 6–9. Light injected with other spatial profiles or powers will exhibit oscillatory behavior. In a MM fiber amplifier seeded with the low-power $LP_{01}$ mode, the transverse spatial profile will adiabatically evolve into that of the fundamental stationary mode as the power is amplified toward $P_{\mathrm{crit}}$ for realistic values of the optical gain.

(2) These conclusions hold whether the fiber is straight or coiled (e.g., for mode-filtering or packaging purposes).

(3) For a given value of $n_2$, $P_{\mathrm{crit}}$ is nearly the same in the bulk material and in a MM step-index fiber, either straight or bent. Other refractive index profiles and small-core fibers require further investigation.

This work was supported by Laboratory Directed Research and Development, Sandia National Laboratories, U.S. Department of Energy, under contract DE-AC04-94AL85000. R. Farrow's e-mail address is farrow@sandia.gov.

### References

1. A. J. Brown, J. Nilsson, D. J. Harter, and A. Tünnermann, eds., Proc. SPIE **6102** (2006).
2. J. P. Koplow, D. A. V. Kliner, and L. Goldberg, Opt. Lett. **25**, 442 (2000).
3. F. Di Teodoro, J. P. Koplow, S. W. Moore, and D. A. V. Kliner, Opt. Lett. **27**, 518 (2002).
4. G. R. Hadley, R. L. Farrow, and A. V. Smith, in Proc. SPIE 6102, 61021S (2006).
5. J. M. Fini, Opt. Express **14**, 69 (2006).
6. E. A. Romanova, L. A. Melnikov, and E. V. Bekker, Microwave Opt. Technol. Lett. **30**, 212 (2001).
7. J. T. Manassah, P. T. Baldeck, and R. R. Alfano, Opt. Lett. **13**, 589 (1988).
8. M. Karlsson, D. Anderson, and M. Desaix, Opt. Lett. **17**, 22 (1992).
9. G. Fibich and F. Merle, Physica D **155**, 132 (2001).
10. M. Igarashi and A. Galvanauskas, in *Conference on Lasers and Electro-Optics*, Vol. 96 of OSA Trends in Optics and Photonics (Optical Society of America, 2004), p. 513.
11. G. Tempea and T. Brabec, Opt. Lett. **23**, 762 (1998).
12. G. Fibich and A. L. Gaeta, Opt. Lett. **25**, 335 (2000).
13. D. Marcuse, Appl. Opt. **21**, 4208 (1982).
14. D. Milam, Appl. Opt. **37**, 546 (1998).